Modelling Subdiffusive Transport in Classical Disordered Systems:

Might this help understand Many-Body Localisation (MBL) observed in quantum disordered systems?

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Introduction

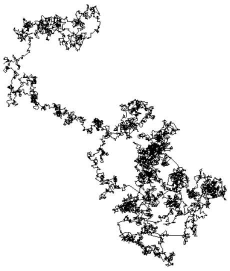

Figure 1: An example of a path traced by a random walker in 2D

The random walker

Imagine you are standing on the centre-spot of a football pitch, and repeatedly toss a fair coin. Every time the coin lands on heads you take a step towards the goal on your left; for tails, your right. This is the most basic model for a random walk. Where might you expect to be after a given number of tosses, n (or equally, if you're tossing at a regular frequency, time)? You'll find that your root mean square (RMS) displacement - or in other words, the width of the region either side of the centre that you expect to have 'explored' - will scale with the square-root of n. This is an example of diffusion: where the transport exponent, α (see Figure 2), is ½.

In this project, a slightly different species of random walk was examined computationally to test whether the addition of disorder – where energy is now required to make a step left or right – might cause the motion to become subdiffusive ($\alpha < ½$) rather than diffusive.

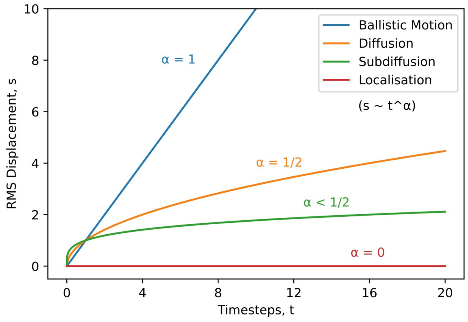

Figure 2: Graph showing different types of motion and how they relate to the transport exponent, α.

Motivation

Many-body localisation

- Unlike classical systems, **a quantum system cannot usually be kept in the same place (or state) indefinitely**. To give the example of a particle trapped in a deep well: this is due to quantum systems' intrinsic property that there will always be a finite probability of the particle 'tunnelling' out of the well, irrespective of its depth.
- MBL is a phenomenon whereby tunnelling is prevented by quantum interference between the different paths by which it could happen. **An MBL system then, as far as we understand, stays where you put it forever**. This offers the only known possible route to keep a quantum system in a certain state indefinitely.
- **This property has potential applications in quantum computing**, where a many-body localised system exposed to some certain random disorder would self-assemble to form qubits, saving painstaking effort.

The link to subdiffusion

- **MBL occurs at the limit where α goes to zero**. The region immediately above this is subdiffusion, so it is reasonable to expect that for a system to become localised, it must first pass through this subdiffusive phase. Understanding the behaviour of these systems as α approaches zero is analogous to understanding what causes them to become localised.
- There are clean (zero disorder) classical models that show subdiffusion, and disordered quantum models. The question is whether there's a link between the origins of subdiffusion in these two cases: a classical disordered model seems a sensible place to start.

Observations

Our Model

- **The dynamic we simulated is similar in principle but different in mechanism** to the simple random walker described above. Imagine that in place of the football pitch we now have, say, 100 baskets (or 'sites') in a line, of which half are occupied at random with a tennis ball and the other half empty. Now, place each basket at a random height between 0 and 2m above the ground.
- Instead of the coin-toss, **we now apply an n-site permutation gate** at a random site along the chain (see Figure 3), which simply shuffles the tennis balls within that window into a new configuration.
- Finally, **the permutation is accepted if the combined heights of the newly occupied baskets fall within an arbitrary height parameter, delta (Δ), of the pre-shuffled total**, and rejected otherwise.
- This process is repeated at every site along the chain, in a random order.

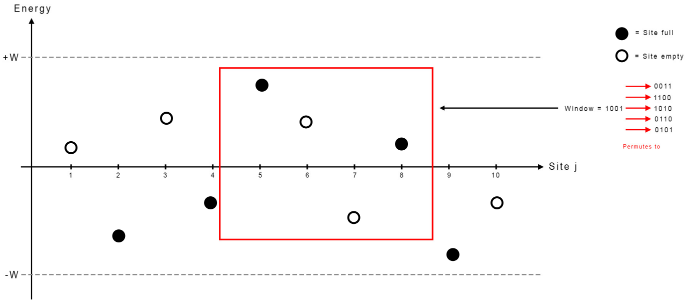

Figure 3: Diagram illustrating the action of the n-site permutation gate (in this case, 4-site), applied at a random site *j* (in this case, site 5). The occupation of the sites is represented by a binary string, where 1 = occupied site and 0 = unoccupied site. The more generic energy scale, between +W and –W, is analogous to height between 0 and 2m above the ground.

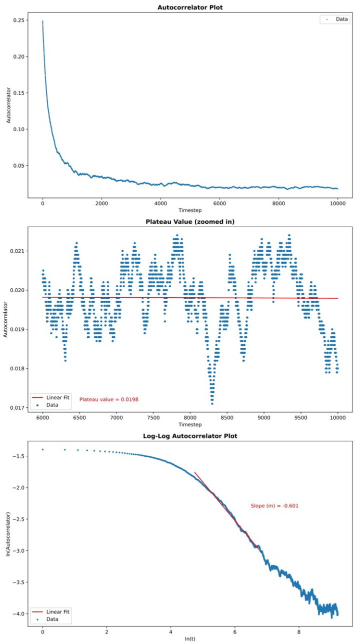

Figure 4: Some data analysis on autocorrelator values run to 10,000 timesteps at $\Delta = 0.1W$: The top graph shows the entire set of values, of which the last 4,000 are seen more closely below, and a linear fit used to establish the plateau value. The lower graph is a log-log plot of all the values, with a fit used to measure the gradient of the linear section. The noise at the bottom-right corner is an artefact of log-log graphs and ignored.

The Data

- **The dynamics were measured using an autocorrelator**, *C*, which is essentially a measure of: if a particle is present on site *j* at time 0, how likely is it still to be there at time t?
- **The plateau value is equal to 1/localisation length** - i.e. the length of the region to which a particle is confined. If this is smaller than the chain length, that implies localisation.
- ***C* also scales with square-root of t**, and since this a power-law relation, this should be seen as a straight line on a log-log plot, as seen at the bottom of Figure 4.

Next Steps

- The algorithm was coded from scratch using Python, and preliminary autocorrelator graphs (such as in Figure 4) generated to confirm the model worked as expected.
- It was later translated into C++ to improve runtime and to allow for running on larger clusters in the future for higher resolution data. The next step will be to examine the model using randomised variable-length gates, which is where we hope to see subdiffusion, rather than the fixed gate lengths used thus far.

Acknowledgements

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